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Multiobjective Optimization of a Port-of-Entry Inspection Policy

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***Abstract*—At the port-of-entry, containers are inspected through a specific sequence of sensor stations to detect the presence of radioactive materials, biological and chemical agents, and other illegal cargo. The inspection policy, which includes the sequence in which sensors are applied and the threshold levels used at the inspection stations, affects the probability of misclassifying a container as well as the cost and time spent in inspection. This work is an extension of a paper by Elsayed *et al.*, which considers an inspection system operating with a Boolean decision function combining station results. In this paper, we present a multiobjective optimization approach to determine the optimal sensor arrangement and threshold levels, while considering cost and time. The total cost includes cost incurred by misclassification errors and the total expected cost of inspection, while the time represents the total expected time a container spends in the inspection system. Examples which apply the approach in various systems are** presented.

***Note to Practitioners*—The inspection of containers at a port-of-entry is becoming a challenging problem as the number of containers increases and a variety of new sensor technologies become available. The sequence of inspections and the level of inspection have a major impact on the total cost of inspection and delay of containers at a port. This paper presents methods that search for the optimum threshold levels at inspection stations, as well as the optimum inspection sequence to minimize the total cost and delays. These methods provide the border protection and customs agencies with approaches based on theoretical foundations yet the results are readily** applicable.

***Index Terms*—Boolean function, multiobjective, probability of false accept, probability of false reject, sensor threshold levels.**

I. Introduction

The significant increase in trade agreements and the growth in the world economy have propelled unprecedented increase in maritime traffic. The value of export goods produced and transported globally in 2000 is about $6.186 trillion [1]. Disruption of such a system has catastrophic consequences on the world economy and our daily needs. In order to minimize sources of disruptions, the United Nations passed several resolutions with the objective of improving security in maritime trade. Likewise, the United States initiated the Container Security Initiative (CSI) to ensure container security through different approaches starting from the origin port of the container and ending at the delivery

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port in the United States. One of these approaches involves the assignment of a "risk" factor associated with containers bound for the U.S. Based on this factor, containers might be subjected to screening at the port of origin and might be "precleared" for importation. Moreover, when containers arrive at United States ports they are subjected to further security inspection. Containers can be randomly selected and subjected to inspection at the port-of-entry (POE). The type of inspections, number of containers to be inspected, and the inspection policy have a profound effect on the cost of the system, risk of accepting undesired containers, and potential delays and congestion at the ports.

In this paper, we consider a POE container inspection system where a fraction of the arriving containers at a port is subjected to a sequence of inspections at different stations. A typical inspection system begins with radiation detection. Containers are driven through a radiation portal monitor (RPM) at approximately 5 miles/h, where radiation emissions are detected. The equipment is passive in that it absorbs radiation from containers as they pass through the RPM. A graphic profile of the radiation reading is produced and if the profile suggests the presence of radioactive material, an alarm is activated. Once an alarm is activated, the container is then subjected to further inspection to determine the source of radiation. This is usually accomplished by using a lightweight handheld radiation isotope identification device (RIID) or an advanced spectroscopic portal (ASP) to identify the radiation isotope. The RIID is more sensitive than the traditional Geiger counter, and takes an isotope reading to determine the type of radiation being emitted. The RIID is capable of distinguishing between naturally occurring, harmless radiation (emitted by materials such as: ceramic tile, granite, kitty litter, fertilizer, or food products containing potassium, including bananas or avocados) and weapons grade radioactive and nuclear materials, including highly enriched uranium or plutonium used in nuclear devices [2].

In addition to inspecting containers for radioactive material and image analysis, the inspection stations may identify biological warfare agents by using biosensors, currently under research and ready for deployment on an experimental basis, that can detect trace amounts of viral or bacterial pathogens *in situ* and provide immediate analysis [3]. Other equipment for detecting biological agents uses fluorescent particle counters for detecting airborne bacteria. In this case the threshold level of the decision is related to the count of these particles. Likewise, methods for detection of chemical agents, currently sarin, cyanide, and pesticides, may be applied at other inspection stations using different sensors [4].

New security improvements at POE require expensive new or retrofitted infrastructure and technology and retrained personnel. New security measures also have major impacts on the cost of container shipping and handling. Both for the container terminal operators and the vessel operators, it is paramount to minimize "turnaround time," i.e., the loading and discharging of containers should be done as quickly as possible [5]. An average container liner spends 60% of its time in port and has a cost of $1000/h or more [6]. Further details of the cost of security for sea cargo transport are given in [7]. Recently, Wein *et al.* [8] state that the hourly waiting cost of a containership arriving at its U.S. port of debarkation is tens of thousands of dollars. To shorten the time spent by vessels, terminal operators need to emphasize resource allocation and reduce the security related inspection processes. Improving security through container inspection has a profound effect on the operations of the shipping supply chain. For example, additional inspection time at a port is likely to lead to a delivery delay [8] which, in turn, tends to increase safety inventories and inventory costs and, hence, reduce supply chain productivity. Reference [8] provides a detailed report of the cost components associated with port security, operation and effect of delay. Clearly, there are two conflicting objectives: improved security and improved productivity; the challenge is to balance both.

Researchers have investigated the problem of container inspection with different objectives. Lewis *et al.* [9] develop the first heuristic procedure to model the problem of moving containers from inbound ships to staging areas where security inspections can occur and moving containers from staging areas and areas where security inspections have been completed to outbound ships. The inspection procedures and sequences have not been considered. Stroud and Saeger [10] consider the problem where a stream of containers arrives at a port and sequential inspections (diagnosis) are conducted to decide whether to pass a container or subject it to further inspection. The container can leave the sequence of inspection stations when some conditions are met or it can continue through other stations until completion of the entire inspection system. Containers that leave the inspection system during inspection can either be accepted or subjected to "manual" inspection. This problem is considered as a binary decision tree. Madigan *et al.* [11] extend the work of Stroud and Saeger by incorporating the threshold levels of the inspection "sensors" and develop a novel binary decision tree search algorithm that operates on a space of potentially acceptable binary decision tree. They describe computationally more efficient approaches for this binary decision tree problem and obtain optimum sensor threshold levels that minimize the total cost of the inspection system.

The problem of determining the optimum inspection sequence has been investigated by many researchers. Since this problem is NP-hard, researchers have focused their investigation on known configured inspection systems such as series, series-parallel, parallel-series, and k-out-of-n systems in order to obtain optimum solutions for a small number of attributes. Lee [12], Raouf *et al.* [13], and Dufuaa and Raouf [14] consider a similar problem for inspection of units in a typical production system where a number of product characteristics are tested, failure of which results in the rejection of the product. The probability that an item will fail each test and the cost associated with the test is known. In some models, tests are not perfect, and there are associated costs of shipping a faulty product to the customer. The problem is to sequence the tests so that the total expected cost is minimized.

Branch and bound and dynamic programming formulations have been proposed to solve the general problem of sequential diagnosis, both of which run in exponential time in general. A dynamic programming approach has been proposed in [15] for threshold functions. Bioch and Ibaraki [16] and Chang *et al.* [17] propose polynomial time algorithms that produce optimal solutions for k-out-of-n system. These algorithms are generalized further in Boros and Unluyurt [18] by providing a generalized algorithm that is optimal for double regular systems (having identical components). Other approaches such as genetic algorithms and artificial intelligence have been utilized. Wein *et al.* [8] model the inspection problem as a game-theoretic approach where game players are the border protection agency (its objective is to maximize probability of detection) and the smugglers of undesired material (its objective is to minimize the probability of detection). Wein *et al.* [19] formulate the inspection problem as a queueing model to determine the optimum number of monitors that maximizes the probability of detection subject to resource constraints.

Elsayed *et al.* [20] present a unique approach to the formulation of the POE inspection problem as an analytical model. Unlike previous work which determines threshold levels and sequence separately, they consider an integrated system and determine them simultaneously. They decompose the POE problem into two subproblems. One problem deals with the determination of the optimum sequence of inspection or the structure of the inspection decision tree in order to achieve the minimum expected inspection cost. This problem is formulated and investigated using approaches parallel to those used in the optimal sequential inspection procedure for reliability systems, as described in [15],

[21]–[27]. The other problem deals with the determination of the optimum thresholds of the inspection stations so as to minimize the cost associated with false reject and false accept.

This paper extends the work in [20] by taking into consideration the time required to complete inspection and treating it as another objective to be minimized. As indicated earlier, the delay in inspection system is a major concern as it has significant economic consequences. Unlike Wein *et al.* [8] where a game-theoretic approach is used, we do not consider the behavior of the smuggler and strategies of the port and focus our work on the development of a new formulation for the POE problem with two objectives: minimization of the total inspection cost which includes cost of inspection and cost of misclassifying containers (acceptable when they are not or unacceptable when they are) and the minimization of delay time. We seek the optimum inspection sequence and the optimum threshold levels of sensors at inspection stations in order to minimize these objectives.

It should be noted that emerging technology will make advanced sensors available to be incorporated into existing inspection systems. These additions will increase the number of stations and complexity of the inspection problem at ports. The methodology proposed in this paper for optimizing container inspection policies may be applied to some current inspection systems; however, it is more practical for the future systems of increased complexity.

This paper is organized as follows. Section II describes the POE container inspection problem. Section III describes the multiple objectives of the optimization problem: the cost of misclassifications, the cost of inspection, and the time spent in inspection. Section IV details approaches for solving the multiobjective problem. Section V presents numerical examples of the methods discussed and finally the last section offers a discussion of the work presented.

II. Problem Description

A. Port-of-Entry (POE) Container Inspection System

In a typical POE, many arriving containers are cleared based on risk scoring or other intelligence information and a small percent (generally 4–5%) are selected for inspection. Here we focus on modeling the inspection of "high-risk" or "untrusted" containers as opposed to the general population. In modeling the POE container inspection system it is assumed that containers arriving for inspection are either inherently acceptable or contain unacceptable materials, and that they have several attributes which may reflect the status (presence or absence) of such material. The inspection system is viewed as a collection of n stations, over which the inspection of a given container is performed sequentially. At each station, a sensor inspects one specific attribute and a pass-or-fail decision is returned (0 or 1, respectively). At each individual station, the decision is dependent on a preset threshold level. Varying this threshold level affects the probability of misclassifying an acceptable container as suspicious or vice versa. The sequence in which stations are to be visited, along with threshold levels to be applied, establishes the inspection policy which is applied to every container arriving for inspection.

The final decision to accept a container or reject it, thereby subjecting the container to further manual inspection (often including a manual "unpacking" method), is based on the decisions at stations and could be decided before all stations have been visited. This container classification is thought of as a system decision function F that assigns to each binary string of decisions $(d_1, d_2, \ldots, d_n)$ a category. In this paper, we focus on the case where there are only two categories. In other words, $F(d_1, d_2, \ldots, d_n) = 0$ indicates negative class and that there is no suspicion with the container and $F(d_1, d_2, \ldots, d_n) = 1$ indicates positive class and that additional inspection is required, usually manual inspection.

In this paper we define, for instance, a system that uses a series Boolean function as applying a decision function F that assigns the container class "1" if any of the individual decisions are fail, $d_i = 1$ for any station i. For example, $F(d_1, d_2, \ldots, d_n) = (d_1 \vee d_2 \vee \cdots d_n)$. This series decision function could be applied in a system where various risks may be indicated by different attributes, and the detection of any one of these attributes warrants further investigation. Furthermore, a system that uses a parallel Boolean function is defined as applying a decision function F that assigns the container the class "1" only if all of the individual decisions are fail. For example, $F(d_1, d_2, \ldots, d_n) = (d_1 \wedge d_2 \wedge \cdots d_n)$. This parallel decision function could be applicable in a system where each attribute is a partial indicator of a particular risk, and only the positive detection of every attribute would be considered significant evidence of unacceptability.

The system decision function to be used depends on the nature of the inspection system and the container attributes being inspected. The decision function does not necessarily represent the physical configuration or layout of the inspection stations but rather the logical flow incorporating station decisions. Conceptual depictions of parallel-series and series-parallel decision functions are given in [20]. The work presented here is designed so that it can be applied with any Boolean function. A few common Boolean decision functions are used in the numerical examples.

B. Modeling of Sensor Measurements

Let x represent true status of a container, and code $x = 1$ if it should be rejected and $x = 0$ if it should be accepted. We assume this container is a sample from a population of interest under which the probability of $x = 1$ is $P(x = 1) = \pi$ and the probability of $x = 0$ is $P(x = 0) = 1 - \pi$.

Let r be the measurement taken by a sensor. This measurement r can, in general, be a numerical (continuous or discrete) reading or a graphical image, as described in Section I. To simplify the presentation of our development and following [10] and [20], we assume

$$\begin{aligned} r &\sim N\left(\mu_0, \sigma_0^2\right) \text{ when } x = 0 \text{ and} \\ r &\sim N\left(\mu_1, \sigma_1^2\right) \text{ when } x = 1 \end{aligned} \tag{1}$$

where $\mu_0 \neq \mu_1$. We choose to use the normal distribution because normally distributed data are the most commonly seen data in practice and it has been used in POE inspection applications [10], [20]. Also, continuous measurements can often be transformed into normally distributed data by the inverse transformation method [28]. Likewise, discrete data sometimes can be well approximated by a normal distribution either by applying the central limit theorem or by some special techniques such as variance stabilization transformation. For instance, as in the example involving particle count in Section I, r_i can be a Poisson count. In this case, its square root transformation $\sqrt{r_i}$ is approximately normal distributed and has often been used in applications [29]. Finally, our formulation can be extended to use any distribution with a calculable cumulative distribution function.

We assume two normal distributions in (1) because we expect to have different sensor readings for a container with true status $x = 1$ and $x = 0$. We also assume that the parameters of the two normal distributions in (1) are known or can be estimated from past inspection history. This model includes a simplistic assumption of the attributes' relation to the true state of an unacceptable container, in which all attribute distributions (not just one) reflect $x = 1$. Note that the task of distinguishing acceptable and unacceptable containers is location and scale invariant to the readings. Without loss of generality, we can assume that $\mu_0 = 0$ and $\mu_1 = 1$. See also [20] for further discussions on this assumption.

C. Threshold Approach

To make a decision d_i based on the sensor measurement r_i at station i, the r_i value is compared against a given threshold value T_i. A fail decision $(d_i = 1)$ is given if the reading r_i is higher than T_i and a pass decision $(d_i = 0)$ if the reading is lower than T_i. The decision d_i at the station level does not always agree with the true status x. There are two types of potential errors: decision $d_i = 1$ when the true status of the container is $x = 0$, and decision $d_i = 0$ when the true status of the container is $x = 1$.

The probability of these two types of errors can be computed by $P(d_i = 1|x = 0) = P(r_i > T_i|x = 0) = 1 - \Phi(T_i/\sigma_{0i})$ and $P(d_i = 0|x = 1) = P(r_i \leq T_i|x = 1) = \Phi((T_i - 1)/\sigma_{1i})$.

D. System Inspection Policy

The minimization of costs associated with performing inspection and misclassification of containers has been formulated in Elsayed *et al.* [20]. Here we expand the optimization objective to include the time required for inspection, which takes into account the effect of delays on the overall system. The performance of the inspection system is determined by both the sequence in which inspection stations are visited and the threshold levels applied at those stations, which we denote collectively as the inspection policy.

Since the optimal parameter values for the cost minimization problem may not minimize time, some compromise may be required. A particular balance of the importance of cost and time may be represented by weights. We consider the case where the relative importance of cost and time is unspecified and therefore we use different importance weights to generate possible solutions that produce a Pareto frontier, as described in Section IV.

III. Performance Measures of Inspection Policy

A. Cost of Misclassification and Inspection

The cost involved in this inspection problem is the sum of any cost incurred as a result of misclassifying a container's status and the actual cost of performing the inspection. As Elsayed *et al.* [20] note, there are two types of misclassification errors at the systems level: falsely rejecting a container that should be cleared and falsely accepting a container that should be rejected. These errors are associated with the probability of false reject (PFR) and the probability of false accept (PFA), respectively. The complementary probabilities of these two errors are true reject (PTR) and true accept (PTA). If D denotes the decision of the entire inspection system of sensors, where $D = 1$ means to reject and $D = 0$ to accept, the four probabilities can be written as follows:

$$\begin{aligned}
\mathrm{PFR} &= P(D = 1|x = 0)\\
\mathrm{PTA} &= P(D = 0|x = 0) = 1 - \mathrm{PFR}\\
\mathrm{PFA} &= P(D = 0|x = 1) \quad \text{and}\\
\mathrm{PTR} &= P(D = 1|x = 1) = 1 - \mathrm{PFA}.
\end{aligned}$$

The inspection decision D depends on the individual inspection results and the system Boolean function. The probability equations just mentioned can be rewritten in terms of the threshold value T_i and σ values related to the inspection station for any given Boolean function. Several examples are given in Elsayed *et al.* [20].

Let c_{FA} be the cost of the system accepting an unacceptable container and c_{FR} be the cost of the system rejecting an acceptable container. Then the total expected cost of system misclassification error per container is $C_F = \pi \mathrm{PFA} c_{FA} + (1 - \pi)\,\mathrm{PFR} c_{FR}$ as described in [20].

The expectation of the cost of inspection C_I is a function of the unit cost to operate each sensor (station) and the probabilities of passing each station. Given a particular set of threshold values, an optimal sequence in which to visit the stations can be found following the conditions in Elsayed *et al.* [20]. The total cost per container arising from misclassification errors and inspection is denoted by $c_{\text{total}} = C_F + C_I$.

B. Time for Inspection

The time required for a container to pass through an inspection station is an important measure of the inspection system performance. It is possible that this time would be related to some characteristic of the inspection station, that is to say the inspection may be sped up or slowed down depending on some operational setting of the sensor. For example, the inspection time may be related to a variable that represents the resolution or other settings of the sensor. Jupp *et al.* [30] report that high statistical profiles are obtained by collecting data for 180 s at each position, i.e., time required to integrate energy and generate profiles for detecting explosives is indirectly related to the threshold level. Therefore, we assume the time t_i spent at a station could be related to the threshold level T_i at that station. For illustrative purposes, we assume in our numerical examples later in Sections IV-B and V the relationship could be an exponential function, expressed as $t_i = a\exp(bT_i)$. Other expressions could also have been used.

To find the total expectation of time spent in the system for a given container we first denote p_i, the probability of passing station i, by

$$\begin{aligned}
p_i = P(d_i = 0) &= \sum_{j=0}^{1} \left[P(d_i = 0|x = j)\, P(x = j)\right]\\
&= (1 - \pi)\,\Phi\left(\frac{T_i}{\sigma_{0i}}\right) + \pi\,\Phi\left(\frac{T_i - 1}{\sigma_{1i}}\right)
\end{aligned}$$

and $q_i = 1 - p_i$ where p_i and q_i are functions of threshold values T_i. Then, the total expected inspection time per container t_{total} can be expressed as $t_{\text{total}} = t_1 + \sum_{i=2}^{n} \left[\prod_{j=1}^{i-1} p_j\right] t_i$, where t_i is the inspection time at station i, for a system with n stations using a series Boolean decision function. For a parallel Boolean decision function, the total expected inspection time per container is $t_{\text{total}} = t_1 + \sum_{i=2}^{n} \left[\prod_{j=1}^{i-1} q_j\right] t_i$.

IV. Multiobjective Optimization

A. Total Expected Cost and Time

As noted in the problem description, we need to determine the optimal design or configuration of sensors in the system and the optimum sets of threshold levels that can achieve the objectives of maximizing inspection system throughput and minimizing the expected total cost. This is a typical multiobjective optimization problem. See, for instance, Eschenauer *et al.* [31], Statnikov and Matuso [32], Fonseca and Fleming [33], [34], and Leung and Wang [35], among others. We formulate the POE problems as a multiobjective optimization problem: $\min_{\text{Sequence},\text{Threshold}}\{c_{\text{total}}, t_{\text{total}}\}$.

There may be a large number or infinite number of optimal solutions in the sense of Pareto-optimality. It is desirable to find as many (optimal) solutions as possible in order to provide more choices to decision makers.

The multiobjective problem is almost always solved by combining the multiple objectives into one scalar objective whose solution is one of the Pareto optimal points for the original problem. A commonly used method to deal with the multiobjective optimization problem is to use the weighted sum approach, where we optimize fitness functions (i.e., weighted sums of the objective functions) for various choices of fixed weights w_1 and w_2, $w_1 + w_2 = 1$.

$$f_{w_1,w_2}(S,T) = w_1 c_{\text{total}} + w_2 t_{\text{total}}.$$

Here, S and T stand for sequence and threshold levels. Thus, the multi-objective optimization problem becomes a sequence of single objective optimization problems, in which we minimize the fitness function for a set of fixed weights w_1 and w_2

$$\min_{S,T} f_{w_1,w_2}(S,T). \tag{2}$$

In this paper, we employ a modified weighted sum approach, in which we utilize some theoretical results to deal with the arrangement of system sequences. Note that the function $f_{w_1,w_2}(S,T)$ is very sensitive due to the discrete nature of the station sequence, in that a switch in the sequence may result in a significant change in the function. The number of arrangements also grows exponentially as the number of inspection stations increase. It is computationally expensive to directly solve the minimization problem in (2). For the system Boolean functions considered in this paper, the optimal sequence can be obtained for a given set of weights and thresholds, as stated in the theorem presented below. So, for a given set of weights and threshold we are able to compute the function

$$f_{w_1,w_2}(T) = \min_{S} f_{w_1,w_2}(S,T) \tag{3}$$

without using an optimization algorithm. In the modified weighted sum approach we, in fact, solve the minimization problem $\min_T f_{w_1,w_2}(T)$.

This modified approach can provide an efficient method to deal with the multiobjective optimization problem under the current context by applying the theorem to select the optimal sequence out of all possible and thus limiting the search area.

Theorem 1:

(a) For a series Boolean decision function, inspecting attributes $i = 1,2,\ldots,n$ in sequential order is optimum, in the sense of minimizing the fitness function for the given set of weights (w_1,w_2) and a given set of thresholds, if and only if

$$\frac{(w_1c_1 + w_2t_1)}{q_1} \le \frac{(w_1c_2 + w_2t_2)}{q_2} \le \cdots \le \frac{(w_1c_n + w_2t_n)}{q_n}$$

(condition 1a).

In this case, the minimal value of the fitness function is given by

$$f_{w_1,w_2}(T) = (w_1c_1 + w_2t_1) + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1} p_j\right](w_1c_i + w_2t_i) + w_1C_F.$$

(b) For a parallel Boolean decision function, inspecting attributes $i = 1,2,\ldots,n$ in sequential order is optimum, in the sense of minimizing the fitness function for the given set of weights (w_1,w_2) and a given set of thresholds, if and only if

$$\frac{(w_1c_1 + w_2t_1)}{p_1} \le \frac{(w_1c_2 + w_2t_2)}{p_2} \le \cdots \le \frac{(w_1c_n + w_2t_n)}{p_n}$$

(condition 1b).

In this case, the minimal value of the fitness function is given by

$$f_{w_1,w_2}(T) = (w_1c_1 + w_2t_1) + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1} q_j\right](w_1c_i + w_2t_i) + w_1C_F.$$

The results in Theorem 1 for series system and parallel system can be extended to systems using parallel-series and series-parallel decision functions, given in Theorem 2. A parallel-series decision function might be useful if each path represents an indicator of one particular risk and a fail decision in every path signifies the presence of that risk. A series-parallel decision function might be useful if each subsystem in series represents a different risk and a fail decision in any subsystem is significant.

Theorem 2:

(a) Consider a parallel-series decision function that has n parallel paths with m sensors in series in each path. If an inspection system with attributes $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$ arranged in parallel-series is optimal, it satisfies the following conditions: the inspection sequence of the series of sensors within each path should be arranged in the order of

$$\frac{(w_1c_{i1} + w_2t_{i1})}{q_{i1}} \le \frac{(w_1c_{i2} + w_2t_{i2})}{q_{i2}} \le \cdots \le \frac{(w_1c_{im} + w_2t_{im})}{q_{im}}$$

and the inspection sequence of parallel paths should be arranged in the order of $F_1/P_1 \le F_2/P_2 \le \cdots \le F_n/P_n$ (condition 2a). Here, F_i and P_i are the (minimal) combined expanse (fitness) of cost and time and the probability of acceptance of the $i\text{th}$ path

$$F_i = (w_1c_{i1} + w_2t_{i1}) + \sum_{j=2}^{m}\left[\prod_{k=1}^{j-1} p_{ik}\right](w_1c_{ij} + w_2t_{ij})$$

and $P_i = P(D_i = 0) = \prod_{j=1}^{m} p_{ij}$. In this case, the minimal value of the fitness function is

$$\begin{aligned} f_{w_1,w_2}(T) &= F_1 + \sum_{i=2}^{n}\left[\prod_{j=1}^{i-1}(1-P_j)\right]F_i + w_1C_F \\ &= F_1 + \sum_{i=2}^{n} F_i \prod_{j=1}^{i-1}\left(1 - \prod_{k=1}^{m} p_{jk}\right) + w_1C_F. \end{aligned}$$

(b) Consider a series-parallel decision function that has n subsystems in series with m sensors in parallel in each subsystem. If an inspection system with attributes $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$ arranged in series-parallel is optimal, it satisfies the following conditions: the inspection sequence of each subsystem should be arranged in the order of $(w_1c_{i1} + w_2t_{i1})/p_{i1} \le (w_1c_{i2} + w_2t_{i2})/p_{i2} \le \cdots \le (w_1c_{im} + w_2t_{im})/p_{im}$ and the inspection sequence of the series of subsystems should be arranged in the order of $F_1/Q_1 \le F_2/Q_2 \le \cdots \le F_n/Q_n$ (condition 2b). Here, F_i and Q_i are the (minimal) combined expanse (fitness) of cost and time and the probability of rejection of the $i\text{th}$ subsystem

$$F_i = (w_1c_{i1} + w_2t_{i1}) + \sum_{j=2}^{m}\left[\prod_{k=1}^{j-1} q_{ik}\right](w_1c_{ij} + w_2t_{ij})$$

and $Q_i = P(D_i = 1) = \prod_{j=1}^{m}(1 - p_{ij})$. In this case, the minimal value of the fitness function is

$$\begin{aligned} f_{w_1,w_2}(T) &= F_1 + \sum_{i=2}^{n}\left(\prod_{j=1}^{i-1} P_j\right)F_i + w_1C_F \\ &= F_1 + \sum_{i=2}^{n} F_i \prod_{j=1}^{i-1}\left\{1 - \prod_{k=1}^{m}(1-p_{jk})\right\} + w_1C_F. \end{aligned}$$

The Appendix shows the proof of the theorems. From the theorems, we describe our modified weighted sum optimization algorithm as follows.

Step 1) Generate a large number, say N, sets of weight pairs (w_1,w_2).

Step 2) For each pair of weights, we solve the minimization problem $T_{\min}^{(w_1,w_2)} = \arg\min_T f_{w_1,w_2}(T)$, where the function $f_{w_1,w_2}(T)$ is computed in a subroutine stated next utilizing the results of the theorems.

Step 3) Obtain the optimal sequence corresponding to $T_{\min}^{(w_1,w_2)}$ (using the results of the theorems) and compute the corresponding optimal throughput time and total cost $(t_{\text{total}}^{(w_1,w_2)}, c_{\text{total}}^{(w_1,w_2)})$.

Step 4) Plot the N pairs of optimal throughput time and cost $(t_{\text{total}}^{(w_1,w_2)}, c_{\text{total}}^{(w_1,w_2)})$, which form the Pareto optimal solutions for the multiobjective optimization problem.

For the parallel and series inspection Boolean systems, we use the following subroutine to calculate the function: $f_{w_1,w_2}(T) = \min_S f_{w_1,w_2}(S,T)$.

1) For each inspection sensor i, calculate $w_1c_i + w_2t_i$.
2) For each inspection sensor i, calculate the ordering criterion $(w_1c_i + w_2t_i)/p_i$ or $(w_1c_i + w_2t_i)/q_i$.
3) Sort the ordering criteria, thus finding the optimal arrangement of sensors, according to the theorems.
4) Calculate the total cost c_{total} and the expected time of inspection t_{total} and return $f = w_1c_{\text{total}} + w_2t_{\text{total}}$.

A similar subroutine can be developed for the series-parallel and parallel-series inspection Boolean systems.

B. Computing Approaches

Standard minimization techniques, such as Newton Raphson type or golden section search and parabolic interpolation algorithms, could perform poorly here due to the discrete nature of the objective function, as discussed in Section IV-A. This is why the modified weighted-sum algorithm is proposed, and a program in MATLAB (The MathWorks, Inc.) is developed to implement it. In Step 2 of the algorithm, the minimization of the function $f_{w_1,w_2}(T)$ is carried out by the built-in MATLAB random search based *ga* function.

A genetic algorithm is an iterative random search algorithm which takes advantage of information in the previous steps (ancestors) to produce new searching points (offspring). It is called a "genetic" algorithm because the principle and design mimic those of genetic evolution found in nature [36]. A genetic algorithm can be applied to solve "a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear" [37].

In the optimization algorithm developed here, the MATLAB function *ga* was used to minimize $f_{w_1,w_2}(T)$ for each pair of weights. Note that the function $f_{w_1,w_2}(T) = \min_S f_{w_1,w_2}(S,T)$ is discrete with regards to T which is inherited from the sequence optimization. Since the optimization function is complex, we compared the results from this method against a complete enumeration method for verification.

We refer to the algorithm using the *ga* function as the GA approach. To verify the results, a grid search method (GS) is implemented. The grid search method does not use either of the developments (theorem or algorithm) in this paper. It is a complete enumeration of possible threshold values and all inspection sequences. A discrete set of threshold values is formed in the range $[0,1]$ using a gradient of 0.05. The total cost and total time are calculated for each possible combination of threshold values and sequence. The resulting cost and time values are plotted and the outermost points along the curve are filtered to represent the solution set that forms the Pareto frontier. Thus the GS method yields a small number of true optimal points compared to the GA method.

The results of the multiobjective optimization are presented in graphical form. The two series plotted in Fig. 1 illustrate the optimal points obtained from the GS and GA methods applied to an inspection system using a parallel Boolean decision function. The system parameters in this example are as follows: $n = 3$, $c = [1\ 1\ 1]$, $\pi = 0.0002$, $\mu_0 = [0\ 0\ 0]$, $\mu_1 = [1\ 1\ 1]$, $\sigma_0 = [0.16\ 0.2\ 0.22]$, $\sigma_1 = [0.3\ 0.2\ 0.26]$, $c_{FA} = 100000$, $c_{FR} = 500$, $a = [20\ 20\ 20]$, $b = [-3\quad -3\quad -3]$, $w_1 = [0 : 0.004 : 1]$, and $w_2 = 1 - w_1$. Square brackets list three specific values corresponding to the three stations in the example. Note that the values of b cause the inspection time to decrease as the threshold level increases.

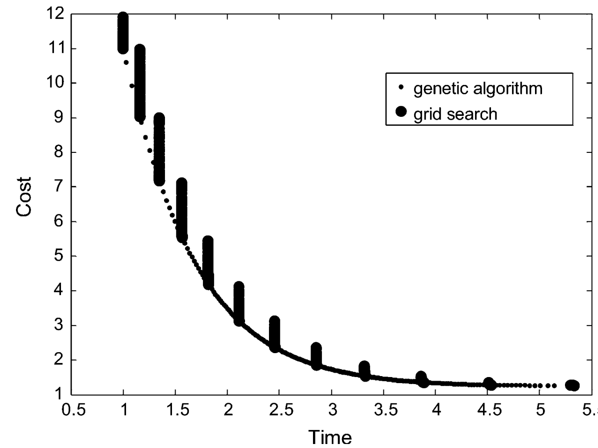


Fig. 1. Comparison of solution methods to multiobjective problem.

TABLE I
EXAMPLES OF PARETO OPTIMAL SOLUTIONS

T_1	T_2	T_3	Sequence	Cost	Time
0.0	0.95	0.05	2-3-1	9.03	1.16
0.0	0.85	0.0	2-1-3	5.54	1.57
0.0	0.75	0.05	2-3-1	3.13	2.11

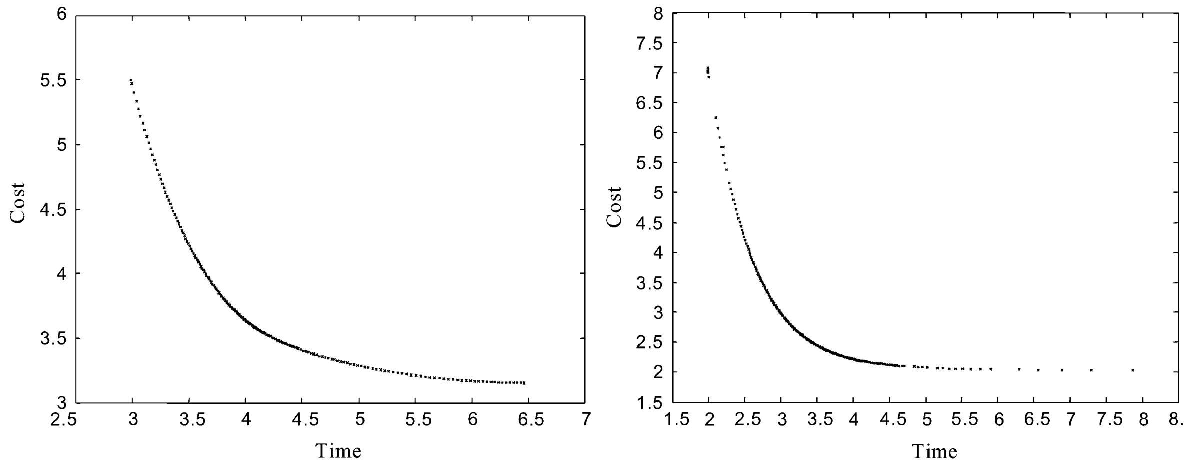
The grid search method produces optimal points that fall into distinct vertical segments due to the discrete nature of the method which only considers T such that $T = 0.05\,\text{m}$, $m = 0, 1, 2, \ldots 20$. The minimum search gradient with an acceptable computation time was used. The series of larger dots contains only the outermost points with respect to the Pareto frontier from this method. Note that only a small number of the points fall on the theoretical Pareto frontier.

The series of smaller dots illustrates the optimal points obtained from the GA method, which seem to include or improve upon the Pareto frontier of solutions with minimal time and cost from the GS method. Each point represents the time and cost for one possible solution, and each solution is defined by a set of threshold values $\{T_i : i = 1, \ldots n\}$ -each to be applied at one of the n inspection stations- and the sequence in which to visit those stations. Table I presents three examples of points chosen from the Pareto frontier of grid search solutions.

It is important to consider program running time in the comparison of methods. The GS method with $\text{grid} = 0.05$ runs in about 6 min, however, only about 12 points of the output are considered to fall within the theoretical Pareto frontier. If the grid is decreased to 0.025, roughly 23 points on the theoretical Pareto frontier are produced but it takes 5 h to run. Further reducing the grid to 0.01 requires more than 200 h to finish. Therefore, it becomes impractical to decrease the grid size in order to generate more optimal points on the theoretical frontier.

The GA method takes about 10.5 h with the current choice of parameters $(\text{PopulationSize} = 80)$ and produces 251 points on the theoretical Pareto frontier. Note that the *ga* function of MATLAB is designed for general purpose use, and we anticipate that the running time can be significantly improved by using a specialized program. Moreover, the GA method produces optimal solutions in all trials that best represent the theoretical Pareto frontier.

The GA method is applied to a system using a series Boolean decision function with the same parameters as the first example. The results



(a) Series Boolean (b) Series-Parallel Boolean

Fig. 2. GA method results.

are presented in Fig. 2(a). It is evident that a change in Boolean function has an effect on the results.

In the third example, the GA method is used to find the multiobjective optimal solution to an inspection problem that uses a series-parallel Boolean function with the system parameters: $m = 2, n = 2$, $c = [1\ 1; 1\ 1]$, $\pi = 0.0002$, $\mu_0 = [0\ 0; 0\ 0]$, $\mu_1 = [1\ 1; 1\ 1]$, $\sigma_0 = [0.16\ 0.2;\ 0.22\ 0.18]$, $\sigma_1 = [0.3\ 0.2;\ 0.26\ 0.18]$, $c_{FA} = 100000, c_{FR} = 500, a = [20\ 20; 20\ 20], b = [-3\quad -3; -3\quad -3]$, $w_1 = [0 : 0.004 : 1]$, and $w_2 = 1 - w_1$.Fig. 2(b) gives the optimal points for this example.

V. Numerical Example

In order to provide some design guidelines for system configuration, we carry out a design of experiment for a system using an $n = 3$ series Boolean decision function. The GA method is utilized in this example.

We specify the following parameters: $\pi, \mu_0, \mu_1, c, \sigma_0, \sigma_1, c_{FA}, c_{FR}$, a, and b. In the design of experiment, we fix $\mu_0 = [0\ 0\ 0]$, $\mu_1 = [1\ 1\ 1]$, $c = [1\ 1\ 1]$, $a = [5\ 5\ 5]$, and $b = [-0.8\ -0.8\ -0.8]$ and consider the four factors π, $\{\sigma_0, \sigma_1\}$, c_{FA} and c_{FR}. Each factor has two or three levels as described below. The set $\{\sigma_0, \sigma_1\}$ is considered as one factor. We now describe the parameters in details.

1) π: Probability of an Unacceptable Container: The probability of a container being unacceptable varies. We assume that two unacceptable containers per 10 000 containers may be appropriate, and choose one fourth of this rate for comparison. Therefore, the two levels of π are 5E-05 and 2E-04.

2) $\{\sigma_0, \sigma_1\}$: Standard Deviations of the Measurements for Acceptable and Unacceptable Containers Respectively: To define this factor, we assume standard deviation values are among {0.15 0.25 0.35}. Furthermore, we choose sets of $\{\sigma_0, \sigma_1\}$ together to affect the area of overlapping probability of the two normal distributions of measurements for each of the three stations. In the following analysis, this overlap is considered as a factor with three levels- small, moderate, and large. In particular, the sets of $\{\sigma_0, \sigma_1\}$ corresponding to the levels are: $\{\sigma_0, \sigma_1\} = \{[0.15\ 0.25\ 0.25],\ [0.25\ 0.15\ 0.15]\}$ small overlap, $\{[0.15\ 0.25\ 0.35],\ [0.35\ 0.25\ 0.15]\}$ moderate overlap, and $\{[0.25\ 0.25\ 0.35],\ [0.35\ 0.35\ 0.25]\}$ large overlap.

3) c_{FA}: Cost of System Accepting an Unacceptable Container: Accepting an unacceptable container has more severe consequences than rejecting an acceptable container. Hence, we use a much higher value for c_{FA} than c_{FR}. We use $1E+05$ as the first level of c_{FA} and $1E+07$ as the second level.

TABLE II
Optimal Solution of Design 19

w_1, w_2	Cost	Time	T_1	T_2	T_3	Seq
0, 1	65.92	6.73	1	1	1	3-2-1
0.25, 0.75	6.21	8.37	0.57	0.75	0.87	3-2-1
0.5, 0.5	5.65	8.63	0.47	0.71	0.91	3-2-1
0.75, 0.25	5.62	8.68	0.46	0.70	0.91	3-2-1
1, 0	5.61	8.71	0.46	0.70	0.91	3-2-1

4) c_{FR}: Cost of System Rejecting an Acceptable Container: Relative to the levels of c_{FA}, we use 200 and 400 as two levels of c_{FR}.

The full $24 (= 2 \times 3 \times 2 \times 2)$ designs are carried out. For each design, we use five weights: $(w_1, w_2) = ((0, 1), (0.25, 0.75), (0.5, 0.5), (0.75, 0.25), (1, 0))$, and use the genetic algorithm and modified weighted sum optimization algorithm in MATLAB to obtain the optimal threshold levels and optimal inspection sequences. As an example, the result of design 19 which uses $\pi = 5E-05, \{\sigma_0, \sigma_1\} = \{[0.15\ 0.25\ 0.35],\ [0.35\ 0.25\ 0.15]\}, c_{FA} = 1E+07$, and $c_{FR} = 200$ is described in Table II.

Given a set of weights, our objective is to minimize $w_1 c_{\text{total}} + w_2 t_{\text{total}}$. Therefore, $(w_1, w_2) = (0, 1)$ corresponds to minimizing total time only and $(w_1, w_2) = (1, 0)$ gives the solution for minimizing total cost only. To gain a general understanding of the effects of four factors in our exploratory analysis, we produce boxplots and use analysis of variance models (ANOVA) to study the outcomes. Since the ranges of total cost and total time for the 24 designs are in similar scale (the range of cost is $[3.329, 20.573]$ and the range of time is $[6.730, 8.894]$) for equal weights $(w_1, w_2) = (0.5, 0.5)$, we use these results for illustration in our analysis. The conclusions from analyses using other choices of the weights are similar.

Fig. 3 shows the boxplots of four factors. The boxplot of factor π indicates that a higher probability of an unacceptable container results in higher cost and time (i.e., higher value of $0.5c_{\text{total}} + 0.5t_{\text{total}}$). The boxplots of c_{FA} and c_{FR} show that higher costs of false decision have higher cost and time. The boxplot of the level of overlap in $\{\sigma_0, \sigma_1\}$ illustrates that larger overlapping distributions result in higher cost and time. In order to see which factors have significant effect, we fit an analysis of variance model and list the results in Table III. Table III shows that the factors of $\{\sigma_0, \sigma_1\}$ and c_{FA} have significant effect under significance level 0.05, while π and c_{FR} do not. These conclusions match well with our intuition. Note that, with a higher level of overlap in the choice of $\{\sigma_0, \sigma_1\}$, the probability of false decision may be greater. Also, higher costs of false decision increase the total cost.

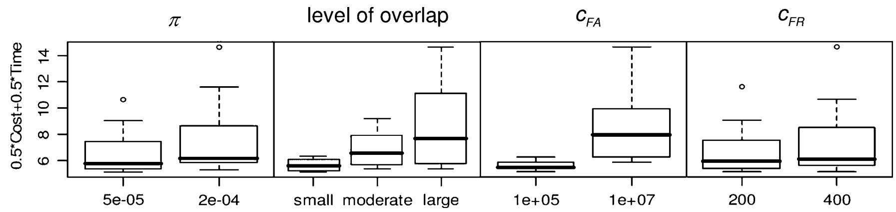


Fig. 3. Boxplots of four factors for equally weighted cost and time.

TABLE III
ANALYSIS OF VARIANCE RESULTS

Factor	Df	Sum Sq	Mean Sq	F value	Pr (>F)
π	1	5.938	5.938	3.009	0.0999
$\{\sigma_0, \sigma_1\}$	2	35.66	17.83	9.036	0.0019
c_{FA}	1	52.135	52.135	26.422	6.86e-05
c_{FR}	1	2.51	2.51	1.272	0.2742

VI. DISCUSSION

This paper investigates and formulates the inspection systems at ports-of-entry. It is formulated as a multiobjective optimization problem that attempts to minimize the total cost, as well as the delay time of the container inspection. The formulation is general and applicable to different systems as the attributes of a typical container are expressed by a Boolean function. The inspection stations in the system can be arranged in series (sequential inspection), parallel, series-parallel, parallel-series, k-out-of-n (where any k stations out of n indicate the presence of undesired attributes) or in any network configuration. Boolean functions corresponding to any of these configurations can be developed. The number of attributes and the inspection sequence have significant impact on the system performance. Likewise, the threshold levels of the sensors are critical in the decision process of accepting or classifying a container as suspicious. They influence the probability of making the "wrong" decision in accepting undesired containers or subjecting acceptable containers to further unneeded inspections.

Based on a numerical study, the proposed weighted sum approach with genetic algorithm is capable of determining the optimum inspection sequence and the threshold levels at each inspection station that result in the optimal system performance measures of cost and time. Another numerical study using design of experiments is carried out and illustrated in a system applying a series Boolean function. It provides a systematic way to study and identify factors that are important in the design of a system. Finally, the multiobjective optimization approach provides Pareto frontier optimal solutions where each solution consists of the optimum sequence of the inspection stations and the corresponding optimum threshold levels. This will enable the decision maker to choose amongst solutions that meet other constraints such as budget, space or layout of the port.

The research focuses on the optimization of an inspection system given current environment and parameters. There are some limitations. For instance, it does not consider the intentions or behaviors of potential smugglers. The information regarding the intention or behaviors of smugglers is often obtained from intelligence gatherings and from studies of past events. This type of information has an impact on the probability parameter π and it could be incorporated in our model, for example the parameter π can be adjusted or estimated based on perceived behavior of the smugglers, as well as other potential factors. Needless to say, POE inspection is a very important and very complex problem. What we have outlined here is part of research trying to provide guidelines to improve the effectiveness and efficiency of the practice of the POE inspection.

APPENDIX
PROOF OF THEOREM 1(a)

For a series Boolean decision function, the fitness function, given the set of weights is

$$\begin{aligned} f_{w_1,w_2}(S,T) &= w_1 c_{\text{total}} + w_2 t_{\text{total}} \\ &= w_1 (C_I + C_F) + w_2 t_{\text{total}} \\ &= w_1 C_I + w_2 t_{\text{total}} + w_1 C_F \\ &= w_1 \left[c_1 + \sum_{i=2}^{n} [\prod_{j=1}^{i-1} p_j]\, c_i \right] \\ &\quad + w_2 \left[t_1 + \sum_{i=2}^{n} [\prod_{j=1}^{i-1} p_j]\, t_i \right] + w_1 C_F \end{aligned}$$

where

$$\begin{aligned} w_1 C_F &= w_1 [\pi \text{PFA} c_{FA} + (1-\pi)\text{PFR} c_{FR}] \\ &= w_1 \begin{bmatrix} \pi \left(\prod_{i=1}^{n} \Phi\left(\frac{T_i - 1}{\sigma_{1i}}\right) \right) c_{FA} \\ +(1-\pi) \left(1 - \prod_{i=1}^{n} \Phi\left(\frac{T_i}{\sigma_{0i}}\right) \right) c_{FR} \end{bmatrix} \end{aligned} \quad (4)$$

and

$$\begin{aligned} w_1 & \left[c_1 + \sum_{i=2}^{n} \left[\prod_{j=1}^{i-1} p_j \right] c_i \right] + w_2 \left[t_1 + \sum_{i=2}^{n} \left[\prod_{j=1}^{i-1} p_j \right] t_i \right] \\ &= (w_1 c_1 + w_2 t_1) + \sum_{i=2}^{n} \left[\prod_{j=1}^{i-1} p_j \right] (w_1 c_i + w_2 t_i). \end{aligned} \quad (5)$$

It is obvious that (4) does not depend on the sequence S, and by [23, Prop. 1], the sequential order is optimum, in the sense of minimizing (5) for the given set of weights (w_1, w_2) and a given set of threshold, if and only if

$$\frac{(w_1 c_1 + w_2 t_1)}{q_1} \le \frac{(w_1 c_2 + w_2 t_2)}{q_2} \le \cdots \le \frac{(w_1 c_n + w_2 t_n)}{q_n}.$$

Theorem 1(b) can be proved by similar argument and [23, Prop. 2] and Theorem 2 can be proved by similar argument and [22, Th. 3.1 and 3.2].

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Surface Patch Reconstruction From "One-Dimensional" Tactile Data

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***Abstract*—This paper studies the reconstruction of unknown curved surfaces through finger tracking. A patch can be generated from tactile data points along three concurrent surface curves under the Darboux frame estimated at the curve intersection point. Surface fitting while minimizing the total (absolute) Gaussian curvature effectively prevents unnecessary folds otherwise expected to result from the use of such "1-D" data. The implementation involves a two-axis joystick sensor, a three-fingered 4-DOF BarrettHand, and a 4-DOF Adept SCARA robot. Experiments have demonstrated good accuracy of reconstruction.**

***Index Terms*—Contour tracking, shape reconstruction, surface fitting, total Gaussian curvature, touch sensing.**

I. Introduction

Objects with curved shapes are ubiquitous in our lives, from small ones such as pens, computer mice, or teapots to big ones such as chairs, cars, or airplanes. The differentiability of a curved shape allows smooth integration of kinematics, dynamics, and control, which paves the way for skillful maneuvers by a robot hand [13].

The human hand can often feel an unknown shape by moving the fingers across its surface. A robot hand with touch sensing capability should be able to accomplish the same. The difference is that the human hand can control its motions smoothly (and freely) based on its sensations. In addition, the number and density of tactels on a finger or the palm of the human hand far exceed what can be fabricated into a tactile array sensor with which a robot hand is equipped.

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